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Fertility, Inequality and Income Growth

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Fertility, Inequality and Income Growth†

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Abstract

This paper sets an endogenous fertility model with a two-sector model: one for the final goods sector and the other for child care service sector. Results of theoretical analysis indicate that the subsidy for children raises the labor share of the child care service sector and that it can increase fertility. An aging population reduces fertility and the labor share of the child care service sector. In addition to these results, we consider monetary policy effects on fertility. Results show that monetary policy can raise fertility and the labor share of the child care service sector by virtue of an increase in the pension benefit if a pay-as-you-go pension exists.

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1. Introduction

This paper sets an endogenous fertility model with a two-sector model: one for the final goods sector and the other for the child care service sector. Results of theoretical analyses indicate that the subsidy for children raises the labor share of the child care service sector and that it can increase fertility. An aging population has consistently declining fertility and labor share of the child care service sector. In addition to these results, we consider monetary policy. Without a pay-as-you-go pension, the monetary policy does not affect fertility or the labor share of the child care service sector. However, if the pay-as-you-go pension exists, then monetary policy affects fertility and the labor share of the child care service sector.

This paper presents derivation of subsidy effects for children. The labor share of the child care service sector increases. Van Groezen, Leers and Meijdam (2003), van Groezen and Meijdam (2008), Fanti and Gori (2009), Yasuoka and Goto (2011), and Yasuoka and Goto (2015) examine child subsidy effects on fertility. Studies described in the related literature do not show the labor share of the child care service sector. This paper can derive the labor share in the endogenous fertility model.

This paper presents consideration of the effect of monetary policy on the fertility. As shown generally, monetary policy stimulates aggregate demand and the gross domestic product (GDP) in the short run. However, in the long run, the monetary policy has neutrality for GDP. By contrast, in the overlapping generations model, the monetary policy has an effect on GDP in the long run. This paper presents derivation that an increase in the monetary stock can raise income growth rate and fertility. It also demonstrates that the labor share of the child care service sector can be affected if a pay-as-you-go pension exists. This result should be referred in consideration of the policy for an aging society with fewer children. Many related papers describe studies of money and inflation (De Gregorio (1993), Mino and Shibata (1995), Yakita (2006), Bhattacharya, Haslag and Martin (2009), Fanti (2012), Chang, Chen and Chang (2013) and Yasuoka (2018a)).

Our argument asserts the importance of monetary policy. In the short run, monetary policy is used as a policy to raise aggregate demand. However, generally speaking, the monetary policy effects vanish in the long run. Our paper presents an overlapping generations model. Then, the effects of monetary policy exist in the long run. Monetary policy raises the inflation rate. Then investment in capital stock increases, representing the so-called the portfolio rebalancing effect. Because of this effect, capital accumulation is facilitated and income growth can be raised.[1]

[1] Based on data from the Cabinet Office, Japan, the monetary stock increases considerably. In 2015,

Finally, fertility increases by virtue of increased income growth. This effect is the same as that of the subsidy policy for children. Our paper describes equivalence between the subsidy for children and monetary policy.

This paper presents an examination of how policy affects wage inequality between the child care service sector and other sectors. The data show that the wage of the child care service sector is lower than that of the average industrial sector.[2] Wage inequality should not be ignored in terms of income inequality. Results derived through these analyses demonstrate that an aging population raises wage inequality between the child care service sector and the final goods sector. The subsidy for child care service and the monetary policy in the model with pension raises fertility and the share of the child care service sector. Thereafter, wage inequality shrinks. The results obtained using the analyses presented in this paper are consistent with those found for a real economy as shown by Fig.1.

[Insert Fig. 1 around here.]

During 2005–2015, social expenditure for the family increased as the child allowance increased. Subsequently fertility and employment for the child care service sector increased. These data are explainable by results obtained using the analyses presented herein. Data of the wage gap separating the child care service sector from all industries are partially inconsistent with results obtained from our analyses. However, during 2005–2011 and during 2013–2015, the wage gap shrank.

Some related papers of the literature examine inequality. Caselli (1999) and Mechl and Zink (2004) set a model of two sectors: one for the sector of the high wage rate and the other for that of the low wage rate. Heterogeneous ability exists among individuals. Because of this setting, the labor share of each sector is determined and income inequality occurs. Kim (1989), Werning (2007) and Aronsson, Sjögren and Dalin (2009) set the model of the individual ability difference and derive optimal taxation system as the redistribution policy. Shindo and Yanagihara (2011) derive that an education subsidy, as a redistributive policy, increases welfare. This paper presents examination of the effects of the subsidy for child care services and monetary policy as the role of redistribution.

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the monetary stock was 906.4 trillion JPY; it reached 974.0 trillion JPY. 7.4% rises. The inflation rate increased by 0.4% during 2015–2017.

[2] Wage inequality between the child care service sector and the average industrial sector exists. Based on data for 2016 (Ministry of Health, Labour and Welfare, Japan), the annual salary of the average industrial sector is 4.899 million JPY. However, the annual salary of the child care service sector is 3.268 Million JPY. The gap is therefore about 1.5 times.

This paper comprises the following contents. Section 2 sets the model; the equilibrium is derived as explained in section 3. Section 4 describes examination of the effect of an aging population on fertility, the labor share, and income growth. Section 5 examines that policy effects on fertility, the labor share, income growth, and inflation in the model without a pension. In section 6, the model with a pension is examined. The final section presents salient conclusions from these analyses.

2. Model

In this model economy, there exist agents of three types: household, firm and government.

2.1 Household

Individuals in the household live in two periods: young and old. Individuals care for the number of children n_t, monetary stock m_t, and consumption in the young and old periods, respectively, as c_{1t}, c_{2t+1}. We assume the following log utility function as

$$u = \alpha ln n_t + \beta ln m_t + \gamma ln c_{1t} + (1 - \alpha - \beta - \gamma)\, ln c_{2t+1} \tag{1}$$

where $0 < \alpha < 1, 0 < \beta < 1, 0 < \gamma < 1$ and $\alpha + \beta + \gamma < 1$.[3]

During the young period, they supply labor inelastically and gain a wage income from which taxes are deducted. They allocate the disposable wage income into child care, consumption when young, and monetary stock and saving. Then, the budget constraint in the young period is shown as

$$(1 - \tau - \varepsilon)\bar{w}_t = (z_t - q_t)n_t + c_{1t} + m_t + s_t. \tag{2}$$

In that equation, τ and ε respectively denote the income tax rate for subsidy policy and the pension. Also, $\bar{w}_t$ denotes the wage income. Furthermore, z_t stands for child care service costs, which can be reduced by the subsidy for children q_t. s_t expresses saving.

During the old period, individuals obtain capital income and a pension benefit. The income obtained during the old period is allocated only to consumption. There is no bequest. The budget constraint in the old period is presented as

$$(1 + r_{t+1})s_t + \frac{m_t}{1 + \pi_{t+1}} + p_{t+1} = c_{2t+1}, \tag{3}$$

where r_{t+1} and π_{t+1} respectively denote the real interest rate and the inflation rate. Older people can obtain pension benefit p_{t+1}.

Considering (2) and (3), we can obtain the following lifetime budget constraint.

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[3] Yakita (2006) sets money in the utility model of Sidrauski (1967) form in the overlapping generations model. We use the utility function assumed by Yakita (2006). Money in the utility model is considered as Walsh (2010).

$$(1-\tau-\varepsilon)\overline{w}_t+\frac{p_{t+1}}{1+r_{t+1}}=(z_t-q_t)n_t+c_{1t}+\left(1-\frac{1}{1+\pi_{t+1}}\frac{1}{1+r_{t+1}}\right)m_t+\frac{c_{2t+1}}{1+r_{t+1}} \tag{4}$$

We consider a two-sector model: one for final goods and the other for the child care services. Based on Caselli (1999), workers must pay training costs to work in the final goods sector. However, no training cost is needed in the childcare service sector. Training cost σw_t is assumed to be distributed uniformly among the workers in $[0,\bar{\sigma}]$.[4] With $w_t(1-\sigma)<w_t^c$, the individual works in the elderly care service sector. Otherwise, the individual works in the final goods sector. Then, the indifference equation is derived as

$$w_t(1-\sigma_t^*)=w_t^c. \tag{5}$$

In that equation, w_t and w_t^c respectively denote the wage rate in the final goods sector and the wage rate in the child care service sector. Therein, σ_t^* is given to hold the (5), with no difference between the final sector and child care sector. Then, $\frac{\sigma^*}{\bar{\sigma}}$ of younger people work in the final goods sector and $\frac{\bar{\sigma}-\sigma^*}{\bar{\sigma}}$ of them work in the child care service sector. Without loss of generality, we set $\bar{\sigma}=1$.

If the household consists of workers in both the final goods sector and the child care service sector, then the lifetime budget constraint can be presented as[5]

$$\begin{aligned}(1-\tau-\varepsilon)\left(\left(\sigma_t^*-\frac{\sigma_t^{*2}}{2}\right)w_t+(1-\sigma_t^*)^2w_t\right)+\frac{p_{t+1}}{1+r_{t+1}}\\ =(z_t-q_t)n_t+c_{1t}+\left(1-\frac{1}{1+\pi_{t+1}}\frac{1}{1+r_{t+1}}\right)m_t+\frac{c_{2t+1}}{1+r_{t+1}}.\end{aligned} \tag{6}$$

Then, considering lifetime budget constraint (6), the optimal allocations to maximize the utility (1) are derived as presented below.

$$n_t=\frac{\alpha\left((1-\tau-\varepsilon)\left(\left(\sigma_t^*-\frac{\sigma_t^{*2}}{2}\right)w_t+(1-\sigma_t^*)^2w_t\right)+\frac{p_{t+1}}{1+r_{t+1}}\right)}{z_t-q_t} \tag{7}$$

$$m_t=\frac{\beta\left((1-\tau-\varepsilon)\left(\left(\sigma_t^*-\frac{\sigma_t^{*2}}{2}\right)w_t+(1-\sigma_t^*)^2w_t\right)+\frac{p_{t+1}}{1+r_{t+1}}\right)}{1-\frac{1}{1+\pi_{t+1}}\frac{1}{1+r_{t+1}}} \tag{8}$$

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4 Yasuoka (2018b) sets the two-sector model: a final goods sector and an elderly care service sector. Yasuoka (2018b) assumes that the elderly care service sector requires no training cost compared to the final goods sector. As might be readily apparent, this training cost is relative. The high training cost means that the person has high ability to work in the elderly care service sector.

5 Noting that $\overline{w}_t=\int_0^{\sigma_t^*}w_t(1-\sigma)\frac{1}{\bar{\sigma}}d\sigma+(1-\sigma_t^*)w_t^c=w_t\left(\sigma_t^*-\frac{\sigma_t^{*2}}{2}\right)+(1-\sigma_t^*)(1-\sigma_t^*)w_t$, one obtains $\left(\sigma_t^*-\frac{\sigma_t^{*2}}{2}\right)w_t+(1-\sigma_t^*)^2w_t$.

$$c_{1t} = \gamma\left((1-\tau-\varepsilon)\left(\left(\sigma_t^* - \frac{\sigma_t^{*2}}{2}\right)w_t + (1-\sigma_t^*)^2 w_t\right) + \frac{p_{t+1}}{1+r_{t+1}}\right) \tag{9}$$

$$c_{2t+1} = (1+r_{t+1})(1-\alpha-\beta-\gamma)\left((1-\tau-\varepsilon)\left(\left(\sigma_t^* - \frac{\sigma_t^{*2}}{2}\right)w_t + (1-\sigma_t^*)^2 w_t\right) + \frac{p_{t+1}}{1+r_{t+1}}\right) \tag{10}$$

2.2 Firm

This model economy includes two sectors of production: the final goods sector and the child care service sector.

The production function in the final goods sector is assumed as

$$Y_t = K_t^{\theta}(A_t L_t)^{1-\theta}, 0 < \theta < 1. \tag{11}$$

Therein, Y_t denotes the output of final goods. The output is inputted by capital stock K_t and labor input L_t. $A_t = a\frac{K_t}{L_t}$ is assumed $(0 < a)$. This model considers the capital stock externality addressed by Romer (1986) and by Grossman and Yanagawa (1993).

With a competitive market and profit maximization, wage rate w_t and real interest rate r_t in the final goods sector are given as

$$w_t = (1-\theta)a^{1-\theta}k_t, \text{ and} \tag{12}$$

$$1 + r_t = \theta a^{1-\theta}, \tag{13}$$

where $k_t = \frac{K_t}{L_t}$. We assume that the capital stock is fully depreciated in a period.

The production function in the child care service sector is assumed as

$$Y_t^c = z_t \rho L_t^c, 0 < \rho, \tag{14}$$

where Y_t^c and L_t^c respectively represent output for child care services and labor input for child care services.[6] Then, defining the profit in the child care service sector as $z_t\rho L_t^c - w_t^c L_t^c$, the wage rate of the child care service sector is given as

$$z_t \rho = w_t^c. \tag{15}$$

Now, we consider N_t as the population size of younger people. Then, the following equations can be derived:

$$L_t = \sigma_t^* N_t, \tag{16}$$

$$L_t^c = (1-\sigma_t^*)N_t. \tag{17}$$

[6] This production function, which includes only labor input, is assumed by Yasuoka and Miyake (2010), Hashimoto and Tabata (2010), and others for studies in which child care or elderly care is examined. Child care and elderly care are considered as labor-intensive services.

2.3 Government

The government provides a subsidy for child care services, which is the same as the child allowance in this model, and a pension benefit.

The child care service subsidy is financed by an income tax, for which the tax rate is τ. With a balanced budget, the budget constraint of the child care service is given as $\tau\left(\left(\sigma_t^* - \frac{\sigma_t^{*2}}{2}\right) + (1-\sigma_t^*)^2\right)w_t = q_t n_t$. Considering $q_t = qw_t$, the budget constraint is given as

$$\tau = \frac{qn_t}{\left(\sigma_t^* - \frac{\sigma_t^{*2}}{2}\right) + (1-\sigma_t^*)^2}. \tag{18}$$

The pension benefit is financed by the income tax levied at the tax rate of ε. This pension system is a pay-as-you-go pension. The budget constraint is set as

$$p_{t+1} = \varepsilon n_t\left(\left(\sigma_t^* - \frac{\sigma_t^{*2}}{2}\right) + (1-\sigma_t^*)^2\right)w_{t+1}. \tag{19}$$

3. Balanced Growth Path

This section presents derivation of the equilibrium in the balanced growth path. Considering (7), (12), (18), and (19), fertility is given as

$$n = \frac{\alpha\left((1-\tau-\varepsilon) + \frac{\varepsilon n(1+g)}{1+r}\right)\left(\left(\sigma^* - \frac{\sigma^{*2}}{2}\right) + (1-\sigma^*)^2\right)}{\frac{1-\sigma^*}{\rho} - q}. \tag{20}$$

In that equation, $1+g$ denotes the income growth rate given as $\frac{k_{t+1}}{k_t}$. Lack of a subscript t shows the variable at the balanced growth path.

With (7), (17) and $\rho L_t^c = n_t N_t$, the market clearing condition of the child care service can be presented as shown below.

$$\frac{\alpha\left((1-\tau-\varepsilon) + \frac{\varepsilon n(1+g)}{1+r}\right)\left(\left(\sigma^* - \frac{\sigma^{*2}}{2}\right) + (1-\sigma^*)^2\right)}{\frac{1-\sigma^*}{\rho} - q} = \rho(1-\sigma^*) \tag{21}$$

Considering capital market equilibrium $K_{t+1} = N_t s_t$ and $s_t = (1-\tau-\varepsilon)\left(\left(\sigma^* - \frac{\sigma^{*2}}{2}\right) + (1-\sigma_t^*)^2\right)w_t - c_{1t} - (z_t - q_t)n_t - m_t$, one can obtain the following income growth equilibrium.

$$1+g = \frac{k_{t+1}}{k_t} = \left(\left(\sigma^* - \frac{\sigma^{*2}}{2}\right) + (1-\sigma^*)^2\right)\times \tag{22}$$

$$\frac{\left(1-\alpha-\frac{\beta}{1-\frac{1}{1+\pi}\frac{1}{1+r}}-\gamma\right)(1-\tau-\varepsilon)(1-\theta)a^{1-\theta}-\frac{\left(\alpha+\frac{\beta}{1-\frac{1}{1+\pi}\frac{1}{1+r}}+\gamma\right)\varepsilon n(1+g)}{1+r}}{\sigma^* n}$$

We consider the equilibrium in the monetary market. Notifying $m_t = \frac{M_t}{P_t N_t}$ (M_t: Nominal monetary stock, P_t: Price index), we can obtain the following equations:

$$\frac{m_{t+1}}{m_t} = 1+g = \frac{1+\mu}{(1+\pi)n}. \tag{23}$$

In that equation, μ denotes the increase rate of nominal monetary stock.

Then, these equations determine income growth rate $1+g$, fertility n, inflation rate $1+\pi$, and labor share σ^* at the balanced growth path. Wage inequality between the final goods sector and the child care service sector can be reduced to

$$\frac{w_t^c}{w_t} = 1-\sigma^*. \tag{24}$$

With a decrease in σ^*, wage inequality shrinks.

4. An Aging Population

This subsection presents our examination of how an aging population affects fertility, income growth, and other characteristics of this economy. As the factor of an aging population, we consider a decrease in the preference for fertility α. In this section, we do not consider the child allowance and pension policy: $\tau = 0$ and $\varepsilon = 0$ are assumed. Then, the fertility and market clearing condition of the child care service sector are

$$n = \frac{\alpha\rho\left(\left(\sigma^* - \frac{\sigma^{*2}}{2}\right) + (1-\sigma^*)^2\right)}{1-\sigma^*}, \tag{25}$$

$$\alpha\left(\left(\sigma^* - \frac{\sigma^{*2}}{2}\right) + (1-\sigma^*)^2\right) = (1-\sigma^*)^2. \tag{26}$$

Considering (25) and (26) and total differentiation with respect to $n, \sigma^*, \alpha,$ we can obtain

$$\frac{d\sigma^*}{d\alpha} = -\frac{\left(\sigma^* - \frac{\sigma^{*2}}{2}\right) + (1-\sigma^*)^2}{(2-\alpha)(1-\sigma^*)} < 0, \tag{27}$$

$$\frac{dn}{d\alpha} = -\rho\frac{d\sigma^*}{d\alpha} > 0. \tag{28}$$

A decrease in the preference for fertility α increases the labor share of the final goods sector and decreases the labor share of the child care service sector. This is an intuitive result. A decrease in demand for child care services reduces the wage rate of the child care service sector. Labor mobility from the child care service sector to the final goods sector occurs. As shown by (28), a decrease in α reduces fertility n. Because of decreased demand for child care, the labor share of the child care service sector decreases.

Effects on income growth and the inflation rate are shown as presented below.

$$\frac{dg}{d\alpha} = -\frac{1+\rho\left(\frac{\alpha(1+g)(2-\sigma^*)}{(1-\theta)a^{1-\theta}(1-\sigma^*)^2} - \frac{\beta(1+r)(1+\pi)}{n\big((1+\pi)(1+r)-1\big)^2}\right)\frac{d\sigma^*}{d\alpha}}{\frac{\rho\sigma^*}{(1-\theta)a^{1-\theta}(1-\sigma^*)} + \frac{\beta(1+r)(1+\pi)}{(1+g)\big((1+\pi)(1+r)-1\big)^2}}, \tag{29}$$

$$\frac{d\pi}{d\alpha} = -(1+\pi)\left(\frac{1}{1+g}\frac{dg}{d\alpha} + \frac{\rho}{n}\frac{d\sigma^*}{d\alpha}\right). \tag{30}$$

The signs of (29) and (30) are ambiguous. The following proposition can be established.

Proposition 1

A decrease in preference for fertility (attributable to an aging population) reduces the labor share of the child care service sector. Wage inequality $\frac{w_t^c}{w_t}$ is magnified.

If $\frac{\alpha(1+g)(2-\sigma^*)}{(1-\theta)a^{1-\theta}(1-\sigma^*)^2} - \frac{\beta(1+r)(1+\pi)}{n\big((1+\pi)(1+r)-1\big)^2} < 0$, which is brought about by a high level of π or a low level of n, then we can obtain $\frac{dg}{d\alpha} < 0$ and $\frac{d\pi}{d\alpha} > 0$: an aging population raises the income growth rate and reduces the inflation rate. The result of the effect of an aging population on the inflation rate is consistent with the case of Japan, where the total fertility rate trend is downward sloping. In two decades, the economy in Japan has remained in a deflationary state, even after its central bank provided sufficient monetary stock. Our model presents the deflationary economy with a decrease in the preference for fertility.

5. Policy without pension

This section presents derivation of how policies of an increase in monetary stock and the subsidy for child care service or a decrease in the preference for fertility as a result of an aging society affect income growth, fertility, and other model components without a pension.

5.1 Monetary Policy

We examine how an increase in μ affects the income growth rate, fertility, and other model parameters. The fertility and the labor share of the child care service sector are given by (25) and (26). Both the fertility and the labor share of the child care service sector are independent of the monetary policy: we can obtain $\frac{dn}{d\mu}=0$ and $\frac{d\sigma^*}{d\mu}=0$.

We next check the effects on the income growth rate and inflation rate. Considering (22) and (23) and total differentiation with respect to g,π,μ we can obtain the following.

$$\frac{dg}{d\mu}=\frac{\dfrac{\beta(1+r)(1+\pi)}{(1+\mu)\big((1+\pi)(1+r)-1\big)^2}}{\dfrac{\rho\sigma^*}{(1-\theta)a^{1-\theta}(1-\sigma^*)}+\dfrac{\beta(1+r)(1+\pi)}{(1+g)\big((1+\pi)(1+r)-1\big)^2}}>0 \tag{31}$$

$$\frac{d\pi}{d\mu}=\frac{\beta(1+r)(1-\theta)a^{1-\theta}(1-\sigma^*)}{\rho\sigma^*\big((1+\pi)(1+r)-1\big)^2}\frac{dg}{d\mu}>0 \tag{32}$$

Then, the following proposition can be established.

Proposition 2

An increase in μ does not affect fertility and the labor share of the child care service sector. The income growth rate and the inflation rate can be pulled up by this policy.

Because of an increase in μ, higher inflation occurs. This result seems to be intuitive. An increase in the inflation rate reduces demand for monetary stock because the cost to hold money increases. Therefore, individuals increase investment in real assets instead of a decrease in demand for monetary stock.

As shown by (25), fertility does not contain either income growth or the inflation rate. However, with pension benefits, fertility depends on income growth. The effect on fertility can change. We examine monetary policy in the model with pension benefit later.

5.2 Subsidy for child care service

This subsection presents an examination of how a subsidy for children means that the child allowance strongly affects fertility, income growth, and other model parameters.

With total differentiation of (20) and (21) with respect to n,σ,τ,q at the approximation of $q=0$, we can obtain the following signs:

$$\frac{d\sigma^*}{dq}=-\frac{\rho(1-\alpha)}{2-\alpha}<0, \tag{33}$$

$$\frac{dn}{d\sigma^*}=-\rho<0. \tag{34}$$

An increase in the subsidy for children raises the labor share of the child care service sector. This is an intuitive result. Because of an increase in the demand for children, the wage rate of the child care service sector increases. Then labor mobility from the final goods sector to the child care service sector occurs. However, an increase in the wage rate of the child care service sector raises the price of child care services. This effect reduces fertility. Finally, this negative effect is smaller than the direct positive effect of the subsidy on fertility: fertility can be pulled up by the subsidy policy.

The effects on income growth and the inflation rate are presented as shown below.

$$\frac{dg}{dq}=-\frac{\dfrac{\alpha\rho\sigma^*(1+g)\left(\dfrac{n}{\left(\sigma_t^*-\frac{\sigma_t^{*2}}{2}\right)+(1-\sigma_t^*)^2}+\dfrac{\rho(1-\alpha)}{1-\sigma^*}\right)}{(1-\theta)a^{1-\theta}}+\dfrac{\beta(1-\sigma^*)(1+r)(1+\pi)}{n\big((1+\pi)(1+r)-1\big)^2}\dfrac{dn}{dq}+\dfrac{\alpha\rho(1+g)}{(1-\theta)a^{1-\theta}(1-\sigma^*)}\dfrac{d\sigma^*}{dq}}{\dfrac{\alpha\rho\sigma^*}{(1-\theta)a^{1-\theta}}+\dfrac{\beta(1-\sigma^*)(1+r)(1+\pi)}{(1+g)\big((1+\pi)(1+r)-1\big)^2}} \tag{35}$$

$$\frac{d\pi}{dq}=-(1+\pi)\left(\frac{1}{1+g}\frac{dg}{dq}+\frac{1}{n}\frac{dn}{dq}\right) \tag{36}$$

Then, the following proposition can be established.

Proposition 3

The subsidy for children increases the labor share of the child care service sector. Fertility increases. The effects on the income growth and the inflation rate are ambiguous. Wage inequality between the two sectors shrinks.

The effects on income growth rate are complicated. The tax burden reduces saving, which consequently reduces income growth. An increase in fertility reduces the income growth effect. These effects are mutually countervailing. Therefore, the effect on income growth is not ambiguous, but effect on the inflation rate is ambiguous.

6. Monetary Policy with Pension

This section presents an examination of how monetary policy affects fertility, the labor share of two sectors and other model parameters. Without a pension, the fertility and labor share of the child care service sector remain unaffected by monetary policy. However, in the model with a pension, fertility and the labor share can be changed by the monetary policy.

Considering (20) and (21), we can obtain the following.

$$\frac{d\sigma^*}{dg} = -\frac{\frac{\alpha\varepsilon n}{1+r}}{\big(2-\alpha(1-\varepsilon)\big)(1-\sigma^*) - \frac{\alpha\varepsilon\rho(1+g)}{1+r}}, \tag{37}$$

$$\frac{dn}{d\sigma^*} = -\rho < 0. \tag{38}$$

As shown by (37), the income growth effect on the labor share of the child care service sector is ambiguous as long as we check the equation. However, we can derive the sign of $\frac{d\sigma^*}{dg}$ as negative.[7] The labor share of the child care service sector increases. Then fertility increases if the monetary policy can raise income growth.

Income growth and the inflation rate are given as

$$\frac{dg}{d\mu} = \frac{\frac{1+\pi}{1+\mu}}{1+\frac{BC(1+\pi)}{A(1+g)}} \tag{39}$$

$$\frac{d\pi}{d\mu} = -\frac{(1+\pi)C}{1+g}\frac{dg}{d\mu} + \frac{1+\pi}{1+\mu} \tag{40}$$

where

$$A = \left(\sigma + \frac{\varepsilon\left(\alpha + \frac{\beta}{1-\frac{1}{(1+r)(1+\pi)}} + \gamma\right)\left(\sigma^* - \frac{\sigma^{*2}}{2} + (1-\sigma^*)^2\right)}{1+r}\right)n$$

$$-\left((1-\sigma^*)\left(1+\frac{\sigma^*(1-\sigma^*)}{\sigma^* - \frac{\sigma^{*2}}{2} + (1-\sigma^*)^2}\right) - \left(\sigma + \frac{\varepsilon\left(\alpha + \frac{\beta}{1-\frac{1}{(1+r)(1+\pi)}} + \gamma\right)\left(\sigma^* - \frac{\sigma^{*2}}{2} + (1-\sigma^*)^2\right)}{1+r}\right)\right)$$

$$\times \frac{\rho(1+g)\frac{\alpha\varepsilon n}{1+r}}{\big(2-\alpha(1-\varepsilon)\big)(1-\sigma^*) - \frac{\alpha\varepsilon\rho(1+g)}{1+r}}$$

$$B = \frac{\beta(1+r)\left((1-\varepsilon)(1-\theta)a^{1-\theta} + \frac{\varepsilon(1+g)}{1+r}\right)\left(\sigma^* - \frac{\sigma^{*2}}{2} + (1-\sigma^*)^2\right)}{(1+\pi)^2(1+r)^2 - 1} > 0$$

[7] See the Appendix for a detailed proof. The model with a pension benefit brings about the dynamics. The Appendix presents dynamics in the model with a pension.

$$C = 1 + \frac{\frac{\rho\alpha\varepsilon}{1+r}}{\left(2-\alpha(1-\varepsilon)\right)(1-\sigma^{*}) - \frac{\alpha\varepsilon\rho(1+g)}{1+r}} > 0$$

With small ε, one can obtain $A > 0$; then $\frac{dg}{d\mu} > 0$ holds. Therefore, the following proposition can be established.

Proposition 4

In the model with the small level of pension, an increase in μ raises the labor share of the child care service sector and fertility because of an increase in income growth.

Fertility depends on monetary policy because monetary policy changes the income growth rate and because the pension benefit depends on the income growth rate if the pension benefit is considered. With a small pension, the income growth rate increases. As shown by (37), the labor share of the child care service sector rises. Then, fertility can be pulled up as shown by (38). An increase in the income growth raises the pension benefit. Subsequently, the lifetime income rises and the demand for child care can be pulled up.

7. Conclusions

This paper sets an endogenous fertility model with monetary policy and examines how the labor share of the final goods sector and the labor share of the child care service sector are changed by monetary policy, a subsidy for child care service, and other model parameters. The subsidy for child care services raises the labor share of the child care service sector and raises fertility. An aging society with a decrease in preference for having children reduces the labor share of the child care service sector and reduces fertility.

Fertility and the labor share of the child care service sector are influenced by monetary policy because of the effects of an increase in the pension benefit. This monetary policy raises the income growth rate. Thanks to an increase in the income growth, the pension benefit increases. Then, demand for child care services and fertility increase.

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Appendix

The sign of $\frac{d\sigma^*}{dg}$ (37)

Defining $L = \alpha\left((1-\varepsilon)\left(\left(\sigma^* - \frac{\sigma^{*2}}{2}\right) + (1-\sigma^*)^2\right) + \frac{\varepsilon n(1+g)}{1+r}\right)$ and $R = \rho(1-\sigma^*)^2$ as shown by (21), respectively, we can depict the following figure. We assume that $\alpha < \rho$. Without this assumption, we can obtain σ^* in the model without pension. Therefore, this assumption must have equilibrium.

[Insert Fig. A.1 around here]

We can obtain the intersection of L and R and show that an increase in g reduces σ^*.

Dynamics in the model with a pay-as-you-go pension

With a pay-as-you-go pension, the dynamics occurs in the model. The dynamics in this model is shown as

$$n_t = \frac{\alpha\rho\left((1-\varepsilon)\left(\sigma_t - \frac{\sigma_t^2}{2} + (1-\sigma_t)^2\right) + \frac{\varepsilon n_t(1+g_t)\left(\sigma_{t+1} - \frac{\sigma_{t+1}^2}{2} + (1-\sigma_{t+1})^2\right)}{1+r}\right)}{1-\sigma_t}. \tag{B.1}$$

$$n_t = \rho(1-\sigma_t). \tag{B.2}$$

$$\begin{aligned} 1+g_t = \frac{1}{\sigma_{t+1}n_t} \times \Bigg(&\left(1-\alpha-\frac{\beta}{1-\frac{1}{1+\pi}\frac{1}{1+r}}-\gamma\right)(1-\varepsilon)(1-\theta)a^{1-\theta}\left(\sigma_t - \frac{\sigma_t^2}{2} + (1-\sigma_t)^2\right) \\ &- \frac{\left(\alpha+\frac{\beta}{1-\frac{1}{1+\pi}\frac{1}{1+r}}+\gamma\right)\varepsilon n_t(1+g_t)\left(\sigma_{t+1} - \frac{\sigma_{t+1}^2}{2} + (1-\sigma_{t+1})^2\right)}{1+r}\Bigg) \end{aligned} \tag{B.3}$$

$$1+\pi_t = \frac{1+\mu}{(1+g_t)n_t}. \tag{B.4}$$

We replace (B.1), (B.3), and (B.4) with the following equations:

$$n_t = n(\sigma_t, \sigma_{t+1}, g_t), \tag{B.5}$$

$$g_t = g(n_t, \pi_t, \sigma_t, \sigma_{t+1}), \tag{B.6}$$

$$\pi_t = \pi(n_t, g_t). \tag{B.7}$$

The values of $\frac{d\sigma_{t+1}}{d\sigma_t}$ are shown as follows.

$$\frac{d\sigma_{t+1}}{d\sigma_t} = \frac{-\rho + \frac{\partial n}{\partial \sigma_t} - \frac{\partial n}{\partial g_t} \frac{\frac{\partial g}{\partial \sigma_t} - \rho\left(\frac{\partial g}{\partial n_t} + \frac{\partial g}{\partial \pi}\frac{\partial \pi}{\partial n_t}\right)}{1 - \frac{\partial g}{\partial \pi}\frac{\partial \pi}{\partial g_t}}}{\frac{\partial n}{\partial \sigma_{t+1}} + \frac{\frac{\partial n}{\partial g_t}\frac{\partial g_t}{\partial \sigma_{t+1}}}{1 - \frac{\partial g}{\partial \pi}\frac{\partial \pi}{\partial g_t}}}. \tag{B.8}$$

The local stability condition is $-1 < \frac{d\sigma_{t+1}}{d\sigma_t} < 1$ in the steady state.

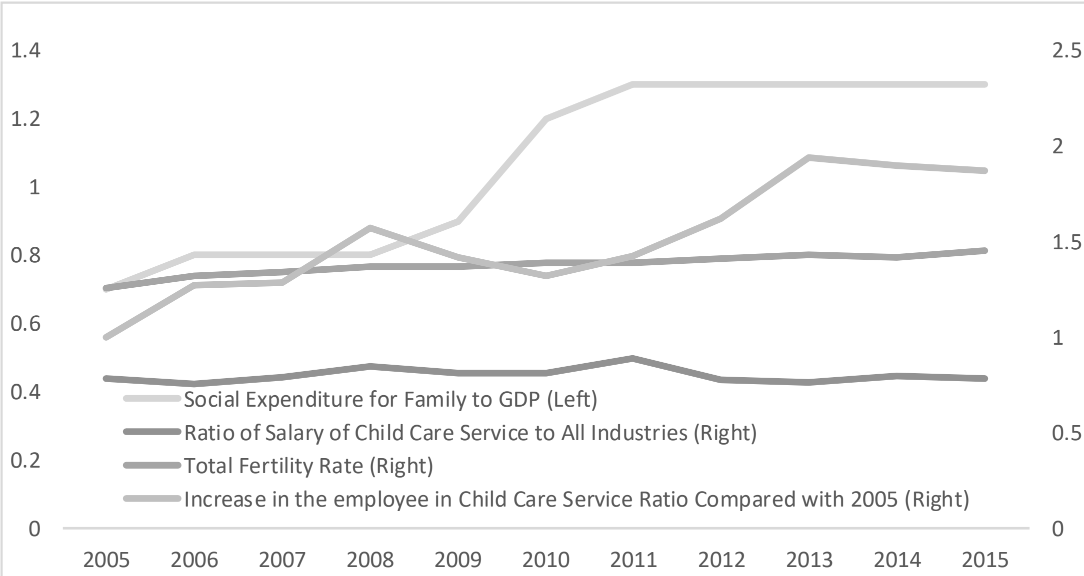

Fig. 1 Fertility and Child Care Service Data in Japan (Data: Ministry of Health, Labour and Welfare 'Demographic statistics', 'Basic Survey on Wage Structure', OECD Statistics 'Public Expenditure on Family by Type of Expenditure (Cash and in Kind), in % GDP'.)

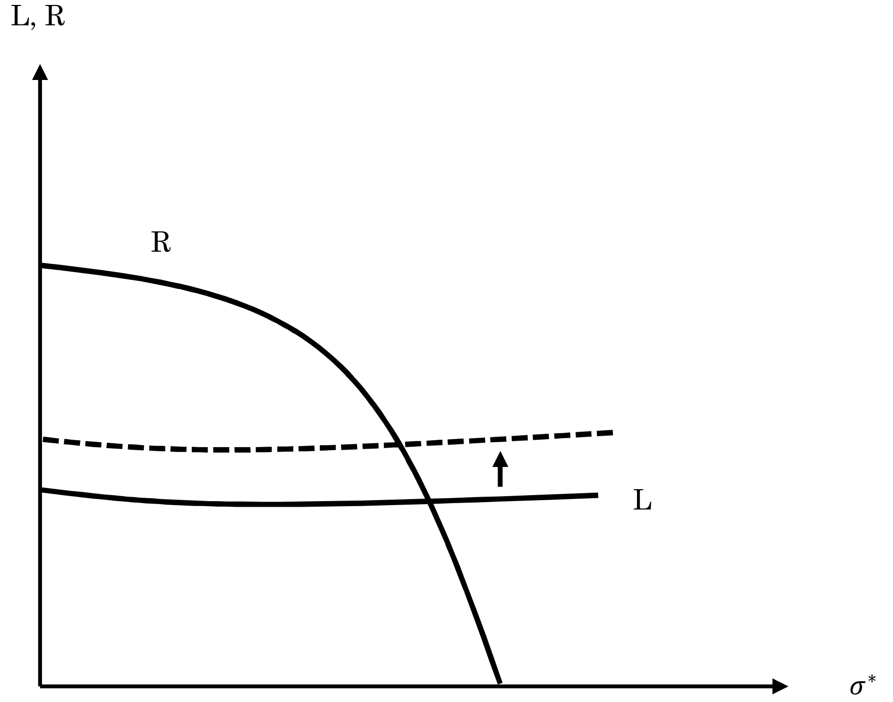

Fig. A.1: Increase in g .